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Quark-lepton flavor democracy and the non-existence of the fourth generation

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Abstract

In the Standard Model with two Higgs doublets (type II), which has a consistent trend to a flavor gauge theory and its related flavor democracy in the quark and the leptonic sectors (unlike the minimal Standard Model) when the energy of the probes increases, we impose the mixed quark-lepton flavor democracy at high “transition” energy and assume the usual see-saw mechanism, and consequently find out that the existence of the fourth generation of fermions in this framework is practically ruled out.

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The number of the light neutrino species with masses below $M_Z/2$ has been measured very accurately at LEP [1], and it is only three. However, we believe this measurement does not completely exclude the existence of the fourth generation, if the neutrino of this generation is, for as yet unknown reasons, very heavy (presumably heavier than $M_Z/2$). In this paper we would like to examine the question of the non-existence of the fourth generation in the framework of flavor gauge theory and its related flavor democracy (FD) [2].

The notion of flavor democracy for quarks (q-q FD) at low (“physical”) energies is well-known by now [3]. It means that a fermion flavor basis¹ exists in which the Yukawa couplings to u_R quarks (and separately to d_R quarks) are all equal and that there is no Cabibbo-Kobayashi-Maskawa (CKM) mixing. The FD in the leptonic sector (charged leptons and Dirac neutrinos) is defined analogously. In reality, fermions at low energies manifest the flavor democratic structure only to a first approximation. However, at increasing energy of probes, the minimal Standard Model (MSM) and the closely related “type I” Standard Model with two Higgs doublets (2HDSM(I)) have a clear trend away from FD, while the usual “type II” Standard Model with two Higgs doublets (2HDSM(II)) has a consistent trend to FD in all quark and leptonic sectors (q-q FD and ℓ - ℓ FD) [2].

In the mass basis, neglecting the light first generation, the trend to FD in the quark sectors (q-q FD) means:

$$\frac{m_s}{m_b}, \frac{m_c}{m_t}, (V_{ckm})_{cb} \longrightarrow 0 \quad \text{as } E \rightarrow \Lambda_{pole}, \quad (1)$$

and analogously in the leptonic sectors (ℓ - ℓ FD). Here, Λ_{pole} is the Landau pole where the Yukawa couplings blow up.

The motivation behind these notions is the following. One could assume that the Standard Model at some high “transition” energy $E_{trans.} \sim \Lambda_{pole}$ is replaced by a new strongly interacting physics with almost flavor-blind forces, where the tiny deviation from the “flavor-blindness” is provided by some as yet unknown mechanism (e.g. a mechanism having its origin in string theory, etc.) and/or possible radiative corrections near $E_{trans.}$. In such a framework, the 2HDSM(II) would be definitely favored as the low energy theory,

¹ This is a basis in which Ψ_L ($\Psi^T = (\psi_u, \psi_d)$) transforms as an iso-doublet under the $SU(2)_L$.

in comparison to the MSM or to its closely related 2HDSM(I) [2]. Furthermore, the Higgs sector could possibly be explained through the mechanism of condensation of the neutral Higgses [4] near $E_{trans.}$, allowing realistically low values for m_t^{phy} ($m_t^{phy} < 200$ GeV for $E_{trans.} \sim \Lambda_{pole} \ll \Lambda_{Planck}$, for a large range of values of the ratio of the vacuum expectation values: $v_u/v_d \lesssim 1.75$ [2],[5]), *unlike* the condensation mechanism leading to the MSM [6]. The entire results concerning the trend to FD can, however, be regarded independently of the motivation outlined above.

If we want to have the number of degrees of freedom at $E_{trans.}$ ($\sim \Lambda_{pole}$) in the Yukawa sector of the 2HDSM(II) additionally reduced, we can *impose* the mixed quark-lepton flavor democracy (q- ℓ FD)

$$m_t \simeq m_{\nu_\tau}^D, \quad m_b \simeq m_\tau \quad \text{at } E \simeq \Lambda_{pole}, \quad (2)$$

which would leave us at high “transition” energy with basically only two Yukawa couplings (g_{up} and g_{down} ; $|g_{up}| \gg |g_{down}|$), and would provide² us with the Dirac neutrino mass $m_{\nu_\tau}^D$ (at $E = 1$ GeV) and Λ_{pole} , for a chosen m_t^{phy} and $\tan \beta \equiv v_u/v_d$. The see-saw mechanism would subsequently furnish us with an estimate (upper bound (*u.b.*)) for the resulting physical neutrino mass [2]

$$M_{Majorana} \gtrsim E_{trans.} \simeq \Lambda_{pole} \implies m_{\nu_\tau}^{phy} \simeq \frac{(m_{\nu_\tau}^D)^2}{M_{Majorana}} \lesssim \frac{(m_{\nu_\tau}^D)^2}{\Lambda_{pole}} = (m_{\nu_\tau}^{phy})^{u.b.}. \quad (3)$$

Here we would like to raise the question whether in this flavor democracy-favored 2HDSM(II) model the imposed q- ℓ FD would be compatible with the existence of the fourth generation of heavy fermions (t', b'), ($\nu_{\tau'}^D, \tau'$):

$$m_{t'} \simeq m_{\nu_{\tau'}}^D, \quad m_{b'} \simeq m_{\tau'} \quad \text{at } E \simeq \Lambda_{pole}. \quad (4)$$

Through the application of the 1-loop renormalization group equations (RGEs) for the Yukawa couplings in the \overline{MS} scheme (see Appendix), this condition (4), together with the known masses of the third generation fermions (we neglect the influence of the light fermions of the first and the second generations - a very good approximation), yield a

² Since $m_b^{phy} \ll m_t^{phy}$, the condition $m_b \simeq m_\tau$ at $E \simeq \Lambda_{pole}$ is automatically satisfied (“pull-up” effect) through the condition $m_t \simeq m_{\nu_\tau}^D$ at $E \simeq \Lambda_{pole}$.

one-to-one correspondence

$$m_t^{phy}, \tan \beta (\equiv \frac{v_u}{v_d}), m_{t'}^{phy}, m_{b'}^{phy} \longleftrightarrow \Lambda_{pole}, m_{\tau'}^{phy}, m_{\nu_{\tau'}}^D, \quad (5)$$

where $m_{\nu_{\tau'}}^D$ is the Dirac mass of the heavy neutrino at $E = 1$ GeV. The meaning of the relation (5) is the following. We choose as an input any specific values of m_t^{phy} , $\tan \beta$, $m_{t'}^{phy}$ and $m_{b'}^{phy}$, and then we look for (“adjust”) $m_{\tau'}^{phy}$ and $m_{\nu_{\tau'}}^D$ ($E = 1$ GeV) such that the q-l FD condition (4) is satisfied, thus obtaining the l.h.s. of (5) as output. The see-saw mechanism would then additionally provide us, similarly as in the case of three generations (cf. eq. (3)), with an estimate (upper bound) for the physical neutrino mass on the r.h.s. of eq. (5)

$$m_{\nu_{\tau'}}^{phy} (\simeq \frac{(m_{\nu_{\tau'}}^D)^2}{M_{Majorana}}) \lesssim \frac{(m_{\nu_{\tau'}}^D)^2}{\Lambda_{pole}} = (m_{\nu_{\tau'}}^{phy})^{u.b.} . \quad (6)$$

These calculations, leading with the q- ℓ FD condition (4) from the l.h.s. to the r.h.s. of (5) (and to (6)), were here performed with the 1-loop RGEs for the Yukawa couplings for the case of four generations (cf. Appendix) in the 2HDSM(II). We considered the first and second generations of fermions as essentially massless and ignored them, i.e. their mixing effects to the heavier fermions, in the RGEs. Furthermore, the threshold for the evolution of the RGEs was taken to be $E_{thresh.} = m_t^{phy}$, i.e. the RGEs for the third and the fourth generation were evolved only for energies $E \geq E_{thresh.} = m_t^{phy}$. For $E < E_{thresh.}$, the physics was considered to be described by the effective theory $SU(3)_c \times U(1)_{em}$ (without Higgs, W, Z, fourth generation and the top) [7], within the corresponding evolution interval $[1 \text{ GeV}, m_t^{phy}]$. We took in the RGEs the 2-loop solution for the $SU(3)_c$ gauge coupling $\alpha_3(E)$, although it turned out that the results do not depend appreciably on it³. For the light third generation masses we took⁴: $m_b^{phy} \simeq 4.3$ GeV, $m_\tau(E = 1 \text{ GeV}) \simeq 1.78$ GeV. To determine the physical masses of heavy quarks,

³ We chose: $\alpha_3(M_Z) = 0.118$, corresponding to $\alpha_3(E = 34 \text{ GeV}) = 0.1387$. The conclusions of this paper do not change if we take the experimentally suggested upper bound $\alpha_3(E = 34 \text{ GeV}) = 0.16$ (see later).

⁴ For simplicity, we chose the third generation Dirac neutrino mass $m_{\nu_\tau}^D(E = 1 \text{ GeV}) = 0$. It turned out that the results are virtually independent of the choice of $m_{\nu_\tau}^D(E = 1 \text{ GeV})$.

we used the (QCD-corrected) relation

$$m_q(E = m_q^{phy}) \simeq \frac{m_q^{phy}}{1 + \frac{4}{3} \frac{\alpha_3(m_q^{phy})}{\pi}} . \quad (7)$$

When calculating the correspondence (5) with the RGEs in this way (taking into account the q - ℓ FD condition (4)), we further impose four physical constraints:

$$m_{t'}^{phy} , m_{b'}^{phy} > m_t^{phy} , \quad (8)$$

$$40 GeV \lesssim m_{\nu_{\tau'}}^{phy} \lesssim \frac{(m_{\nu_{\tau'}}^D)^2}{\Lambda_{pole}} , \quad (9)$$

$$\text{Yukawa couplings at electroweak scale are within perturbative range} , \quad (10)$$

$$(\Delta\rho)_{heavyfermions (h.f)} < 0.0076 . \quad (11)$$

The first two constraints are based on experimental data. The third constraint was imposed to ensure that the Standard Model considered here (2HDSM(II)) is not manifestly non-perturbative⁵. Specifically, we chose for (10) a rather generous constraint:

$$m_{t'}^{phy} , m_{b'}^{phy} \lesssim 0.5 \Lambda_{pole} . \quad (12)$$

The 1-loop expression for the contributions to $\Delta\rho$ from heavy fermions (in \overline{MS} scheme) is [8]

$$\begin{aligned} (\Delta\rho)_{h.f.} = & \frac{G_F \sqrt{2}}{16\pi^2} \left\{ 3K_{qcd} [m_t^2 + m_{t'}^2 + m_{b'}^2 - \frac{2m_{t'}^2 m_{b'}^2}{(m_{t'}^2 - m_{b'}^2)} \ln \frac{m_{t'}^2}{m_{b'}^2}] \right. \\ & \left. + [m_{\tau'}^2 + m_{\nu_{\tau'}}^2 - \frac{2m_{\tau'}^2 m_{\nu_{\tau'}}^2}{(m_{\tau'}^2 - m_{\nu_{\tau'}}^2)} \ln \frac{m_{\tau'}^2}{m_{\nu_{\tau'}}^2}] \right\} , \end{aligned} \quad (13)$$

where m_t , $m_{t'}$, $m_{b'}$, $m_{\tau'}$, $m_{\nu_{\tau'}}$ are the physical masses, and K_{qcd} is the QCD correction parameter [9]

$$K_{qcd} \simeq 1 - \frac{(2\pi^2 + 6)}{9\pi} \alpha_3 . \quad (14)$$

The fourth constraint (11) was obtained from an essentially model-independent analysis of the LEP data [10]. Furthermore, the analysis of the experimental evidence from $B - \bar{B}$

⁵ At $E \simeq (0.7 - 0.75)\Lambda_{pole}$, the 2-loop contribution to the r.h.s. of the RGEs (cf. Appendix) for the Yukawa couplings acquires approximately the same magnitude as the 1-loop contribution - this estimate is based on the structure of the 2-loop RGEs for the MSM.

and $D - \bar{D}$ mixing, Δm_K , ϵ_K and missing E_T measurements at $p\bar{p}$ colliders suggests bounds on $\tan\beta$ [11]

$$0.5 \lesssim \tan\beta (\equiv \frac{v_u}{v_d}) \lesssim 10. \quad (15)$$

It is interesting that in the case of *three* generations we had obtained $\tan\beta$ restricted between 0.53 and 2.1, once we imposed the q-l FD at Λ_{pole} , the see-saw condition (3) and took the experimentally prejudiced values $(m_{\nu_\tau}^{phy})^{u.b.} < 31$ MeV and $m_t^{phy} = (175 \pm 20)$ GeV [2],[5].

The mass of the top quark has recently been measured by the CDF group at Fermilab [12]: $m_t^{phy} \simeq (174 \pm 17)$ GeV. Therefore, we first took in our calculations $m_t^{phy} = 160$ GeV, and varied $\tan\beta$. When imposing the first three constraints (8)-(10) on the obtained results, we derived the allowed regions in the $m_{b'}^{phy}$ vs. $m_{t'}^{phy}$ plane, as depicted in Figs. 1, 2 and 3 for the cases $\tan\beta = 0.33$, 1 and 3.5, respectively. Some remarks are in order here. The lower (dotted) boundaries in the figures originate from the heavy neutrino constraint (9), the upper (full line) boundaries from the “perturbative” constraint (10). When further varying $\tan\beta$, we can obtain the possible region:

$$0.24 < \tan\beta < 5.2, \quad (\text{for } m_t^{phy} = 160 \text{ GeV})$$

where the perturbative constraint (12) does not allow us to push $\tan\beta$ beyond the above interval, since it then rules out the entire plane $m_{b'}^{phy}$ vs. $m_{t'}^{phy}$ (for $m_{t'}^{phy}, m_{b'}^{phy} > m_t^{phy} = 160$ GeV). Hence, the entire $\tan\beta$ -interval as permitted by the constraints (8)-(10) is only within $[0.24, 5.2]$.

One surprising feature of Figs. 1, 2, 3 is that the “perturbative” constraint (10) in both cases of $\tan\beta = 0.33$ (Fig. 1) and 3.5 (Fig. 3) turns out to be more restrictive than in the “middle” case of $\tan\beta = 1$ (Fig. 2). This is due to the fact that the Yukawa coupling $g_{t'}$ (, $g_{b'}$) at low energies ($E \simeq m_t^{phy}$) is quite large⁶ in the case $\tan\beta = 0.33$ (, 3.5, respectively), and consequently it increases with energy rather quickly and Λ_{pole} is relatively small ($\Lambda_{pole} \lesssim 10^4$ GeV). In the case of $\tan\beta = 1$, on the other hand, both $g_{t'}$ and $g_{b'}$ are relatively small at low energies and consequently increase relatively slowly with energy, so that the “perturbative” constraint (10) is not as restrictive.

⁶ $g_{t'}$ (, $g_{b'}$) at energies near Λ_{pole} is the dominant quark Yukawa coupling in the cases of $\tan\beta < 1.2$ (, > 1.3 , respectively), for $m_{t'}^{phy} \approx m_{b'}^{phy}$

The most surprising thing is that, when we impose in addition the “ ρ -constraint” (11) upon the regions in Figs. 1, 2, 3, these regions become ruled out entirely. Namely, in the depicted regions the calculations yield⁷: $(\Delta\rho)_{h.f.}^{min} = 0.0086, 0.0155$, and 0.0084 , for $\tan\beta = 0.33, 1$ and 3.5 , respectively, all exceeding the maximum allowed value 0.0076 . This turns out to be true for the entire interval $[0.24, 5.2]$ for $\tan\beta$: $(\Delta\rho)_{h.f.} \geq 0.0081$. The largest contribution to the values of $(\Delta\rho)_{h.f.}^{min}$ comes from m_t^{phy} (0.0072), the rest predominantly from heavy leptons. In the cases $\tan\beta \geq 3.5$, the minima for $(\Delta\rho)_{h.f.}$ are reached at the “perturbative” bound (where $m_{t'}^{phy} \simeq 0.5\Lambda_{pole}$). All in all, the $\Delta\rho$ -constraint (11) cannot be satisfied in the regions allowed by the other three constraints. Consequently, in the discussed case of $m_t^{phy} = 160$ GeV, the q- ℓ FD condition (4) in the flavor democracy–favored 2HDSM(II) model, together with the usual see-saw mechanism, effectively rules out the existence of the fourth generation.

When choosing higher m_t^{phy} (> 160 GeV), $(\Delta\rho)_{h.f.}^{min}$ becomes even bigger and the “perturbatively allowed” interval for $\tan\beta$ becomes narrower. Hence, we can argue that for $m_t^{phy} \geq 160$ GeV the q- ℓ FD requirement at $E \simeq E_{trans.} \simeq \Lambda_{pole}$ in the flavor democracy–favored 2HDSM(II) is *not* compatible with the existence of the fourth generation, i.e. this framework would practically rule out the fourth generation. We also performed calculations at the experimental lower bound for m_t^{phy} (for $m_t^{phy} = 155$ GeV). The “perturbatively allowed” interval for $\tan\beta$ is now slightly larger: $[0.23, 5.37]$. However, the $(\Delta\rho)_{h.f.}^{min}$, in the regions allowed by the first three conditions, is in this case still above 0.0076 , just reaching this value at $\tan\beta = 0.23, 5.37$, as seen from Fig. 4.

If we used instead of the rather conservative “perturbative” constraint (12) a more restrictive one (i.e. by replacing there $0.5\Lambda_{pole}$ by a smaller value), we would exclude the existence of the 4th generation in the described framework even for the cases of m_t^{phy} lighter than 155 GeV. Finally, we also investigated the effect of the CKM mixing by introducing $V_{tb} \approx V_{\nu_\tau\tau} \approx 0.2$ (at low energy), instead of no 3-4 generation mixing, and we found that the results changed only for a fraction of one percent. Therefore, the CKM mixing does not influence the results of this paper.

⁷ when taking for the QCD parameter $\alpha_3(E = 34 \text{ GeV})$ the experimentally suggested upper bound 0.16 , the values of $(\Delta\rho)_{h.f.}^{min}$ increase slightly, but less than one percent.

We conclude that for all $m_t^{phy} \geq 155$ GeV, the existence of the fourth generation within the described scenario (q-l FD at Λ_{pole} , and see-saw mechanism) in the flavor democracy-favored 2HDSM(II) model is practically ruled out. We also note that if we abandon the assumption of the see-saw mechanism, we cannot rule out the existence of the fourth generation within the discussed flavor democracy framework. Namely, the low energy masses of τ' and of the Dirac $\nu_{\tau'}$ are always above 100 GeV and rather close to each other, and we can choose such $m_{t'}^{phy} \approx m_{b'}^{phy}$ that the contributions of the fourth generation fermions to $(\Delta\rho)_{h.f.}$ are practically zero, thus resulting in having no effects of the $\Delta\rho$ -constraint.

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Appendix: “Type II Standard Model with two Higgs doublets and its RGEs”

In the 2HDSM(II), only one Higgs doublet ($H^{(u)}$) couples to the “up-type” right-handed fermions (f_{uR}) and is hence responsible for their masses, and analogously the other Higgs doublet ($H^{(d)}$) couples solely to the f_{dR}

$$\begin{aligned} \mathcal{L}_{Yukawa} = & - \sum_{i,j=1}^3 \{ [(\bar{q}_L^{(i)} \tilde{H}^{(u)}) q_{uR}^{(j)} U_{ij}^{(q)} + \text{h.c.}] + [(\bar{q}_L^{(i)} H^{(d)}) q_{dR}^{(j)} D_{ij}^{(q)} + \text{h.c.}] \} \\ & - \sum_{i,j=1}^3 \{ [(\bar{l}_L^{(i)} \tilde{H}^{(u)}) l_{uR}^{(j)} U_{ij}^{(\ell)} + \text{h.c.}] + [(\bar{l}_L^{(i)} H^{(d)}) l_{dR}^{(j)} D_{ij}^{(\ell)} + \text{h.c.}] \} , \end{aligned} \quad (1)$$

where $U^{(q)}$, $D^{(q)}$, $U^{(\ell)}$, $D^{(\ell)}$ are 4×4 Yukawa matrices in flavor basis (in the case of four generations), and we use the notation

$$\begin{aligned} H^{(\alpha)} &= \begin{pmatrix} H^{(\alpha)+} \\ H^{(\alpha)o} \end{pmatrix}, \quad \tilde{H}^{(\alpha)} = i\tau_2 H^{(\alpha)*}, \quad \langle H^{(\alpha)} \rangle_o = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\alpha \end{pmatrix} \quad (\alpha = u, d) . \\ q^{(i)} &= \begin{pmatrix} q_u^{(i)} \\ q_d^{(i)} \end{pmatrix}, \quad q^{(1)} = \begin{pmatrix} u \\ d \end{pmatrix}, \quad q^{(2)} = \begin{pmatrix} c \\ s \end{pmatrix}, \quad q^{(3)} = \begin{pmatrix} t \\ b \end{pmatrix}, \quad q^{(4)} = \begin{pmatrix} t' \\ b' \end{pmatrix}, \\ l^{(i)} &= \begin{pmatrix} l_u^{(i)} \\ l_d^{(i)} \end{pmatrix}, \quad l^{(1)} = \begin{pmatrix} \nu_e^D \\ e \end{pmatrix}, \quad l^{(2)} = \begin{pmatrix} \nu_\mu^D \\ \mu \end{pmatrix}, \quad l^{(3)} = \begin{pmatrix} \nu_\tau^D \\ \tau \end{pmatrix}, \quad l^{(4)} = \begin{pmatrix} \nu_{\tau'}^D \\ \tau' \end{pmatrix}. \end{aligned} \quad (2)$$

The corresponding 1-loop renormalization group equations (RGEs) for the “squared” Yukawa matrices $Q^{(u)}$ ($= U^{(q)} U^{(q)\dagger}$), $Q^{(d)}$ ($= D^{(q)} D^{(q)\dagger}$), $L^{(u)}$ ($= U^{(\ell)} U^{(\ell)\dagger}$) and $L^{(d)}$ ($= D^{(\ell)} D^{(\ell)\dagger}$), in \overline{MS} scheme are

$$\begin{aligned} 32\pi^2 \frac{d}{dt} Q^{(u)} &= 3Q^{(u)^2} - \frac{3}{2}(Q^{(u)} Q^{(d)} + Q^{(d)} Q^{(u)}) + 2Q^{(u)}(\Xi_u^{(q)} - A_u^{(q)}), \\ 32\pi^2 \frac{d}{dt} Q^{(d)} &= 3Q^{(d)^2} - \frac{3}{2}(Q^{(u)} Q^{(d)} + Q^{(d)} Q^{(u)}) + 2Q^{(d)}(\Xi_d^{(q)} - A_d^{(q)}), \\ 32\pi^2 \frac{d}{dt} L^{(u)} &= 3L^{(u)^2} - \frac{3}{2}(L^{(u)} L^{(d)} + L^{(d)} L^{(u)}) + 2L^{(u)}(\Xi_u^{(\ell)} - A_u^{(\ell)}), \\ 32\pi^2 \frac{d}{dt} L^{(d)} &= 3L^{(d)^2} - \frac{3}{2}(L^{(u)} L^{(d)} + L^{(d)} L^{(u)}) + 2L^{(d)}(\Xi_d^{(\ell)} - A_d^{(\ell)}), \end{aligned} \quad (3)$$

where

$$\begin{aligned}
t &= \ln \left(\frac{2E^2}{v^2} \right) , \\
\Xi_u^{(q)} &= \Xi_u^{(\ell)} = \text{Tr}(3Q^{(u)} + L^{(u)}) , \\
\Xi_d^{(q)} &= \Xi_d^{(\ell)} = \text{Tr}(3Q^{(d)} + L^{(d)}) , \\
A_u^{(q)} &= \pi \left[\frac{17}{3} \alpha_1 + 9\alpha_2 + 32\alpha_3 \right] , \quad A_d^{(q)} = A_u^{(q)} - 4\pi\alpha_1 , \\
A_u^{(\ell)} &= \pi[3\alpha_1 + 9\alpha_2] , \quad A_d^{(\ell)} = \pi[15\alpha_1 + 9\alpha_2] .
\end{aligned} \tag{4}$$

Here, E is the energy of probes, α_1 , α_2 and α_3 are the usual Standard Model gauge couplings corresponding to $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively, with $N_g = 4$ in the case of 4 generations.

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Figure Captions:

FIG. 1. The region in the $m_{b'}^{phy}$ vs. $m_{t'}^{phy}$ plane, as allowed by the three constraints (8)-(10). The lower (dotted) boundary originates from the heavy neutrino constraint (9), the other boundary (full line) from the “perturbative” constraint (10) (i.e. (12)). The figure is for the 2HDSM(II), with q-l FD at Λ_{pole} , $m_t^{phy} = 160$ GeV, and $\tan \beta (\equiv v_u/v_d) = 0.33$.

FIG. 2. As Fig. 1, but for $\tan \beta = 1$.

FIG. 3. As Fig. 1, but for $\tan \beta = 3.5$

FIG. 4. The $\Delta\rho$ -contribution from heavy fermions, i.e. its minimal value $((\Delta\rho)_{h.f.}^{min})$ in the regions of the $m_{b'}^{phy}$ vs. $m_{t'}^{phy}$ plane that are allowed by the three constraints (8)-(10), as function of $\tan \beta (\equiv v_u/v_d)$. The full line is for $m_t^{phy} = 160$ GeV, the dotted for $m_t^{phy} = 155$ GeV. Both lines are above the maximal allowed value of 0.0076 (cf. (11)). The figure is for the 2HDSM(II) with four generations, see-saw, and q-l FD at Λ_{pole} .

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